The second boundary condition can be interpreted as *in the general solution, replace all the x's by L and T(x) by T_2*. That is,

$$T(L) = C_1L + C_2 \quad \rightarrow \quad T_2 = C_1L + T_1 \quad \rightarrow \quad C_1 = \frac{T_2 - T_1}{L}$$

Substituting the C_1 and C_2 expressions into the general solution, we obtain

$$T(x) = \frac{T_2 - T_1}{L}x + T_1 \qquad \textbf{(2–56)}$$

which is the desired solution since it satisfies not only the differential equation but also the two specified boundary conditions. That is, differentiating Eq. 2–56 with respect to x twice will give d^2T/dx^2, which is the given differential equation, and substituting $x = 0$ and $x = L$ into Eq. 2–56 gives $T(0) = T_1$ and $T(L) = T_2$, respectively, which are the specified conditions at the boundaries.

Substituting the given information, the value of the temperature at $x = 0.1$ m is determined to be

$$T(0.1\text{ m}) = \frac{(50 - 120)°\text{C}}{0.2\text{ m}}(0.1\text{ m}) + 120°\text{C} = \mathbf{85°C}$$

(*b*) The rate of heat conduction anywhere in the wall is determined from Fourier's law to be

$$\dot{Q}_{\text{wall}} = -kA\frac{dT}{dx} = -kAC_1 = -kA\frac{T_2 - T_1}{L} = kA\frac{T_1 - T_2}{L} \qquad \textbf{(2–57)}$$

The numerical value of the rate of heat conduction through the wall is determined by substituting the given values to be

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (1.2\text{ W/m·K})(15\text{ m}^2)\frac{(120 - 50)°\text{C}}{0.2\text{ m}} = \mathbf{6300\ W}$$

Discussion Note that under steady conditions, the rate of heat conduction through a plane wall is constant.

Boundary condition:

$$T(0) = T_1$$

General solution:

$$T(x) = C_1x + C_2$$

Applying the boundary condition:

$$\underset{\underset{T_1}{\underbrace{0}}}{\underset{\uparrow}{T(x)}} = C_1 \underset{\underset{0}{\uparrow}}{x} + C_2$$

Substituting:

$$T_1 = C_1 \times 0 + C_2 \rightarrow C_2 = T_1$$

It cannot involve x or $T(x)$ after the boundary condition is applied.

FIGURE 2–42

When applying a boundary condition to the general solution at a specified point, all occurrences of the dependent and independent variables should be replaced by their specified values at that point.

EXAMPLE 2–14 Heat Loss through a Steam Pipe

Consider a steam pipe of length $L = 20$ m, inner radius $r_1 = 6$ cm, outer radius $r_2 = 8$ cm, and thermal conductivity $k = 20$ W/m·K, as shown in Fig. 2–49. The inner and outer surfaces of the pipe are maintained at average temperatures of $T_1 = 150°C$ and $T_2 = 60°C$, respectively. Obtain a general relation for the temperature distribution inside the pipe under steady conditions, and determine the rate of heat loss from the steam through the pipe.

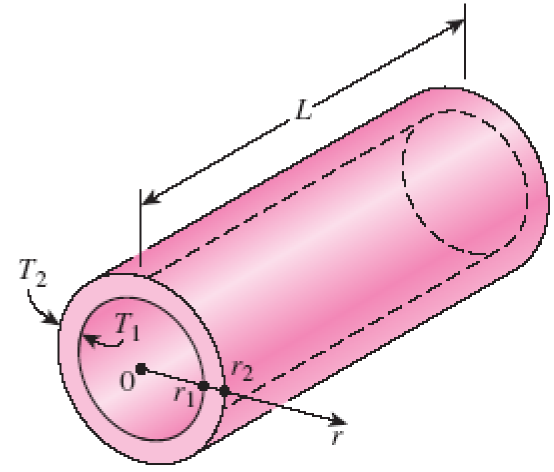


FIGURE 2–49

Schematic for Example 2–14.

SOLUTION A steam pipe is subjected to specified temperatures on its surfaces. The variation of temperature and the rate of heat transfer are to be determined.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction, and thus $T = T(r)$. **3** Thermal conductivity is constant. **4** There is no heat generation.

Properties The thermal conductivity is given to be $k = 20$ W/m·K.

Analysis The mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

with boundary conditions

$$T(r_1) = T_1 = 150°\text{C}$$
$$T(r_2) = T_2 = 60°\text{C}$$

Integrating the differential equation once with respect to r gives

$$r\frac{dT}{dr} = C_1$$

where C_1 is an arbitrary constant. We now divide both sides of this equation by r to bring it to a readily integrable form,

$$\frac{dT}{dr} = \frac{C_1}{r}$$

Again integrating with respect to r gives (Fig. 2–50)

$$T(r) = C_1 \ln r + C_2 \quad \text{(a)}$$

We now apply both boundary conditions by replacing all occurrences of r and $T(r)$ in Eq. (*a*) with the specified values at the boundaries. We get

$$T(r_1) = T_1 \quad \rightarrow \quad C_1 \ln r_1 + C_2 = T_1$$
$$T(r_2) = T_2 \quad \rightarrow \quad C_1 \ln r_2 + C_2 = T_2$$

which are two equations in two unknowns, C_1 and C_2. Solving them simultaneously gives

$$C_1 = \frac{T_2 - T_1}{\ln(r_2/r_1)} \quad \text{and} \quad C_2 = T_1 - \frac{T_2 - T_1}{\ln(r_2/r_1)} \ln r_1$$

Substituting them into Eq. (*a*) and rearranging, the variation of temperature within the pipe is determined to be

$$T(r) = \frac{\ln(r/r_1)}{\ln(r_2/r_1)}(T_2 - T_1) + T_1 \quad \text{(2–58)}$$

The rate of heat loss from the steam is simply the total rate of heat conduction through the pipe, and is determined from Fourier's law to be

$$\dot{Q}_{\text{cylinder}} = -kA\frac{dT}{dr} = -k(2\pi rL)\frac{C_1}{r} = -2\pi kLC_1 = 2\pi kL\frac{T_1 - T_2}{\ln(r_2/r_1)} \quad \text{(2–59)}$$

The numerical value of the rate of heat conduction through the pipe is determined by substituting the given values

$$\dot{Q} = 2\pi(20 \text{ W/m·K})(20 \text{ m})\frac{(150 - 60)°\text{C}}{\ln(0.08/0.06)} = \mathbf{786 \text{ kW}}$$

Discussion Note that the total rate of heat transfer through a pipe is constant, but the heat flux $\dot{q} = \dot{Q}/(2\pi rL)$ is not since it decreases in the direction of heat transfer with increasing radius.

EXAMPLE 2–15 **Heat Conduction through a Spherical Shell**

Consider a spherical container of inner radius $r_1 = 8$ cm, outer radius $r_2 = 10$ cm, and thermal conductivity $k = 45$ W/m·K, as shown in Fig. 2–51. The inner and outer surfaces of the container are maintained at constant temperatures of $T_1 = 200°C$ and $T_2 = 80°C$, respectively, as a result of some chemical reactions occurring inside. Obtain a general relation for the temperature distribution inside the shell under steady conditions, and determine the rate of heat loss from the container.

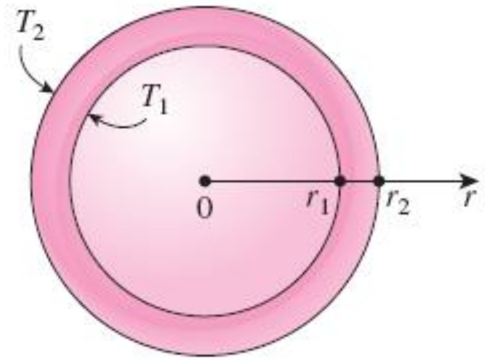


FIGURE 2–51

Schematic for Example 2–15.

SOLUTION A spherical container is subjected to specified temperatures on its surfaces. The variation of temperature and the rate of heat transfer are to be determined.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint, and thus $T = T(r)$. **3** Thermal conductivity is constant. **4** There is no heat generation.

Properties The thermal conductivity is given to be $k = 45$ W/m·K.

Analysis The mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$

with boundary conditions

$$T(r_1) = T_1 = 200°\text{C}$$
$$T(r_2) = T_2 = 80°\text{C}$$

Integrating the differential equation once with respect to r yields

$$r^2\frac{dT}{dr} = C_1$$

where C_1 is an arbitrary constant. We now divide both sides of this equation by r^2 to bring it to a readily integrable form,

$$\frac{dT}{dr} = \frac{C_1}{r^2}$$

Again integrating with respect to r gives

$$T(r) = -\frac{C_1}{r} + C_2 \tag{a}$$

We now apply both boundary conditions by replacing all occurrences of r and $T(r)$ in the relation above by the specified values at the boundaries. We get

$$T(r_1) = T_1 \quad \rightarrow \quad -\frac{C_1}{r_1} + C_2 = T_1$$

$$T(r_2) = T_2 \quad \rightarrow \quad -\frac{C_1}{r_2} + C_2 = T_2$$

which are two equations in two unknowns, C_1 and C_2. Solving them simultaneously gives

$$C_1 = -\frac{r_1 r_2}{r_2 - r_1}(T_1 - T_2) \qquad \text{and} \qquad C_2 = \frac{r_2 T_2 - r_1 T_1}{r_2 - r_1}$$

Substituting into Eq. (a), the variation of temperature within the spherical shell is determined to be

$$T(r) = \frac{r_1 r_2}{r(r_2 - r_1)}(T_1 - T_2) + \frac{r_2 T_2 - r_1 T_1}{r_2 - r_1} \tag{2–60}$$

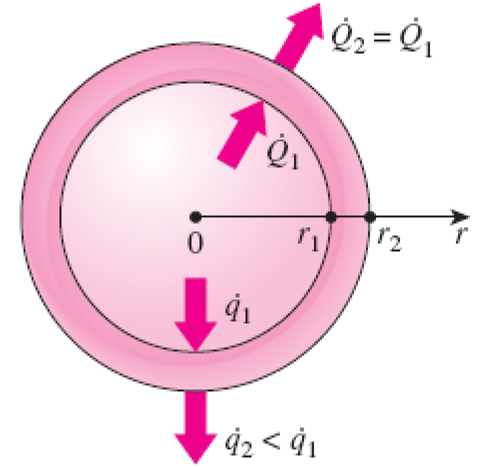
The rate of heat loss from the container is simply the total rate of heat conduction through the container wall and is determined from Fourier's law

$$\dot{Q}_{\text{sphere}} = -kA\frac{dT}{dr} = -k(4\pi r^2)\frac{C_1}{r^2} = -4\pi k C_1 = 4\pi k r_1 r_2 \frac{T_1 - T_2}{r_2 - r_1} \tag{2–61}$$

The numerical value of the rate of heat conduction through the wall is determined by substituting the given values to be

$$\dot{Q} = 4\pi(45\ \text{W/m}\cdot\text{K})(0.08\ \text{m})(0.10\ \text{m})\frac{(200 - 80)°\text{C}}{(0.10 - 0.08)\ \text{m}} = \mathbf{27.1\ kW}$$

Discussion Note that the total rate of heat transfer through a spherical shell is constant, but the heat flux $\dot{q} = \dot{Q}/4\pi r^2$ is not since it decreases in the direction of heat transfer with increasing radius as shown in Fig. 2–52.



$$\dot{q}_1 = \frac{\dot{Q}_1}{A_1} = \frac{27.1\ \text{kW}}{4\pi(0.08\ \text{m})^2} = 337\ \text{kW/m}^2$$

$$\dot{q}_2 = \frac{\dot{Q}_2}{A_2} = \frac{27.1\ \text{kW}}{4\pi(0.10\ \text{m})^2} = 216\ \text{kW/m}^2$$

FIGURE 2–52

During steady one-dimensional heat conduction in a spherical (or cylindrical) container, the total rate of heat transfer remains constant, but the heat flux decreases with increasing radius.

HEAT GENERATION IN A SOLID

Many practical heat transfer applications involve the conversion of some form of energy into *thermal* energy in the medium.

The process involve *internal heat generation*, which explain by a rise in temperature throughout the medium.

Some examples of heat generation are

- *resistance heating* in wires,

- *chemical reactions* in a solid, and

- *nuclear reactions* in nuclear fuel rods

where electrical, chemical, and nuclear energies are converted to heat, respectively.

Heat generation in an electrical wire of outer radius r_o and length L can be expressed as

$$\dot{e}_{\text{gen}} = \frac{\dot{E}_{\text{gen, electric}}}{V_{\text{wire}}} = \frac{I^2 R_e}{\pi r_o^2 L} \qquad (\text{W/m}^3)$$

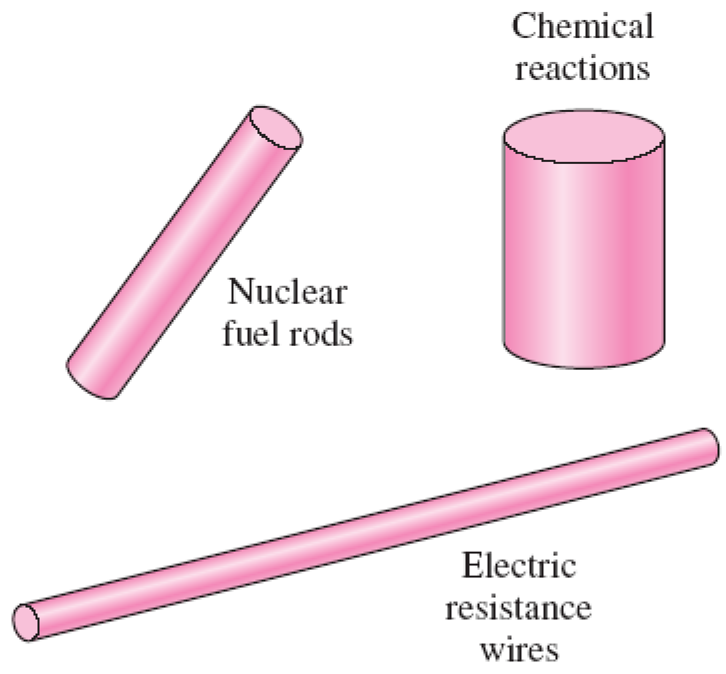


FIGURE 2–53

Heat generation in solids is commonly encountered in practice.

The main parameters involve are surface temperature T_s and the maximum temperature T_{max} that occurs in the medium in *steady* operation. Thus, the energy balance become:

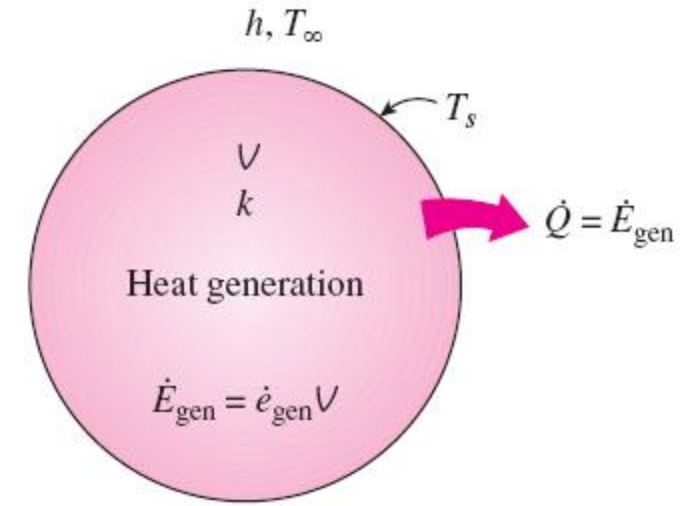


FIGURE 2–54

At steady conditions, the entire heat generated in a solid must leave the solid through its outer surface.

$$\begin{pmatrix} \text{Rate of} \\ \textit{heat transfer} \\ \text{from the solid} \end{pmatrix} = \begin{pmatrix} \text{Rate of} \\ \textit{energy generation} \\ \text{within the solid} \end{pmatrix}$$

$$\dot{Q} = \dot{e}_{gen}V \qquad \text{(W)}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) \qquad \text{(W)}$$

Thus,

$$T_s = T_\infty + \frac{\dot{e}_{gen}V}{hA_s}$$

Eq. 2.66

For a large *plane wall* of thickness $2L$ ($A_s = 2A_{wall}$ and $V = 2LA_{wall}$) with both sides of the wall maintained at the same temperature T_s, a long solid *cylinder* of radius r_o ($A_s = 2\pi r_o L$ and $V = \pi r^2{}_o L$), and a solid *sphere* of radius r_o ($A_s = 4\pi r_o^2$ and $V = \frac{4}{3}\pi r_o^3$), Eq. 2–66 reduces to

$$T_{s,\,\text{plane wall}} = T_\infty + \frac{\dot{e}_{gen}L}{h} \qquad T_{s,\,\text{cylinder}} = T_\infty + \frac{\dot{e}_{gen}r_o}{2h} \qquad T_{s,\,\text{sphere}} = T_\infty + \frac{\dot{e}_{gen}r_o}{3h}$$

When inner medium of cylinder generate heat and transfer the heat to the outer surface as shown in Fig. 2-55

$$-kA_r \frac{dT}{dr} = \dot{e}_{gen} V_r$$

where $A_r = 2\pi rL$ and $V_r = \pi r^2 L$

become
$$-k(2\pi rL)\frac{dT}{dr} = \dot{e}_{gen}(\pi r^2 L) \quad \rightarrow \quad dT = -\frac{\dot{e}_{gen}}{2k} r dr$$

Integrating from $r = 0$ where $T(0) = T_0$ to $r = r_o$ where $T(r_o) = T_s$ yields

$$\Delta T_{\text{max, cylinder}} = T_0 - T_s = \frac{\dot{e}_{gen} r_o^2}{4k}$$

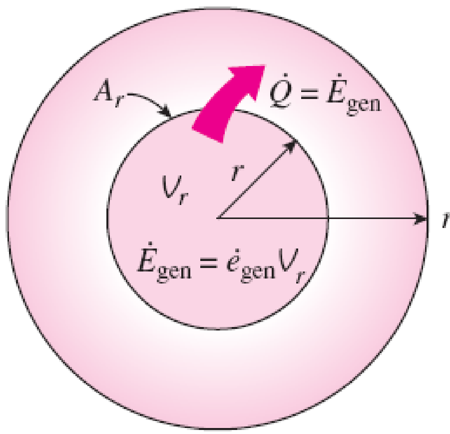


FIGURE 2–55
Heat conducted through a cylindrical shell of radius r is equal to the heat generated within a shell.

Where T_0 is centerline temperature of cylinder, which is the maximum temperature

When ΔT_{max} is available, the centerline temperature can be determined.

$$T_{center} = T_0 = T_s + \Delta T_{max}$$

For plane wall

$$\Delta T_{max,\ plane\ wall} = \frac{\dot{e}_{gen} L^2}{2k}$$

and sphere

$$\Delta T_{max,\ sphere} = \frac{\dot{e}_{gen} r_o^2}{6k}$$

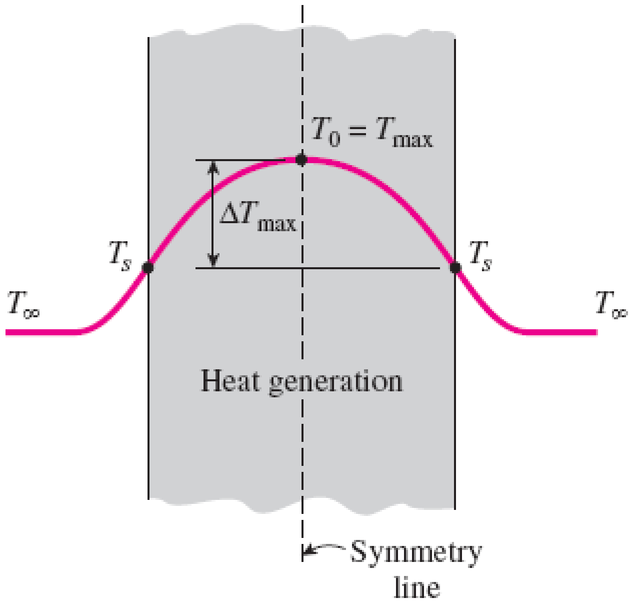


FIGURE 2–56

The maximum temperature in a symmetrical solid with uniform heat generation occurs at its center.

VARIABLE THERMAL CONDUCTIVITY, *k*(*T*)

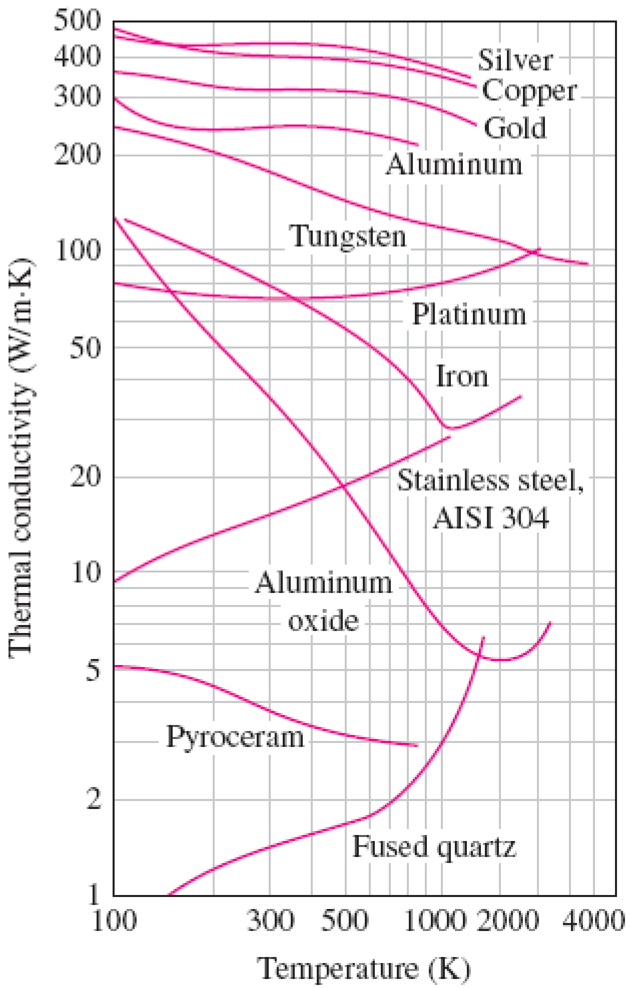


FIGURE 2–62

Variation of the thermal conductivity of some solids with temperature.

When the variation of thermal conductivity with temperature in a specified temperature interval is large, it may be necessary to account for this variation to minimize the error.

When the variation of thermal conductivity with temperature $k(T)$ is known, the average value of the thermal conductivity in the temperature range between T_1 and T_2 can be determined from

$$k_{\text{avg}} = \frac{\int_{T_1}^{T_2} k(T)dT}{T_2 - T_1}$$

$$\dot{Q}_{\text{plane wall}} = k_{\text{avg}} A \frac{T_1 - T_2}{L} = \frac{A}{L} \int_{T_2}^{T_1} k(T)dT$$

$$\dot{Q}_{\text{cylinder}} = 2\pi k_{\text{avg}} L \frac{T_1 - T_2}{\ln(r_2/r_1)} = \frac{2\pi L}{\ln(r_2/r_1)} \int_{T_2}^{T_1} k(T)dT$$

$$\dot{Q}_{\text{sphere}} = 4\pi k_{\text{avg}} r_1 r_2 \frac{T_1 - T_2}{r_2 - r_1} = \frac{4\pi r_1 r_2}{r_2 - r_1} \int_{T_2}^{T_1} k(T)dT$$

The variation in thermal conductivity of a material with temperature can be approximated as a linear function and expressed as

$$k(T) = k_0(1 + \beta T)$$

β temperature coefficient of thermal conductivity.

The *average* value of thermal conductivity in the temperature range T_1 to T_2 in this case can be determined from

$$k_{avg} = \frac{\int_{T_1}^{T_2} k_0(1 + \beta T)dT}{T_2 - T_1} = k_0\left(1 + \beta\frac{T_2 + T_1}{2}\right) = k(T_{avg})$$

The *average thermal conductivity* in this case is equal to the thermal conductivity value at the *average temperature.*

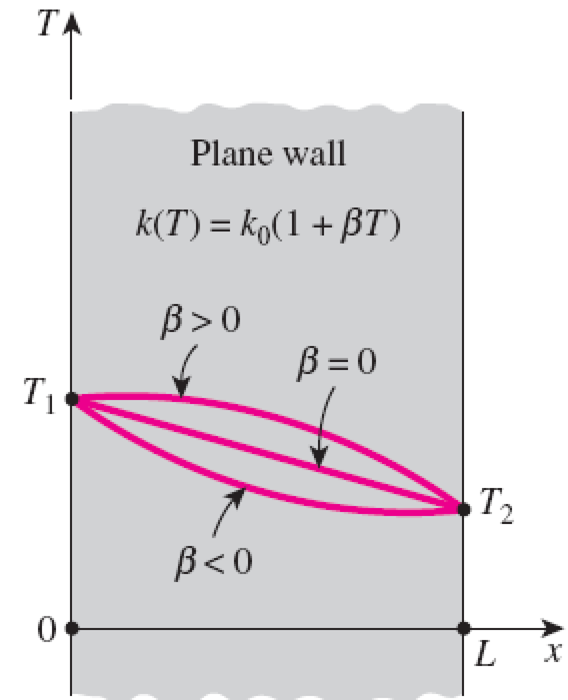
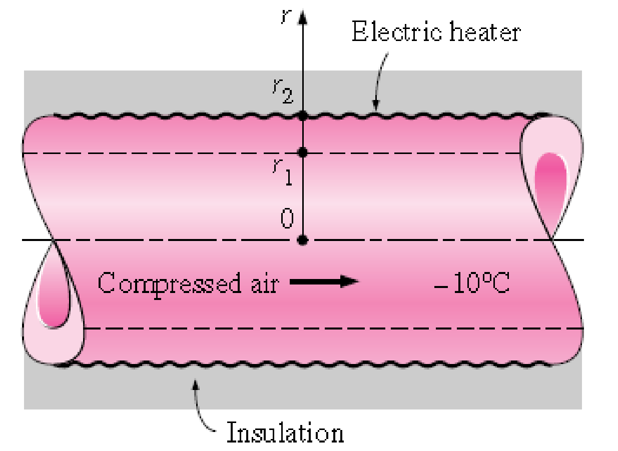


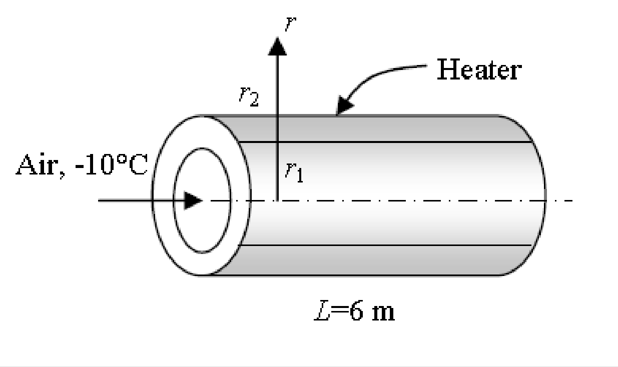
FIGURE 2–63

The variation of temperature in a plane wall during steady one-dimensional heat conduction for the cases of constant and variable thermal conductivity.

Problem 1

When a long section of a compressed air line passes through the outdoors, it is observed that the moisture in the compressed air freezes in cold weather, disrupting and even completely blocking the air flow in the pipe. To avoid this problem, the outer surface of the pipe is wrapped with electric strip heaters and then insulated. Consider a compressed air pipe of length L= 6 m, inner radius r_1 =3.7 cm, outer radius r_2= 4.0 cm, and thermal conductivity k=14 W/m·°C equipped with a 300-W strip heater. Air is flowing through the pipe at an average temperature of -10°C, and the average convection heat transfer coefficient on the inner surface is h=30 W/m^2·°C. Assuming 15 percent of the heat generated in the strip heater is lost through the insulation, (a) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the pipe, (b) obtain a relation for the **variation of temperature** in the pipe material by solving the differential equation, and (c) evaluate the inner and outer surface temperatures of the pipe.





$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

$$-k\frac{dT(r_1)}{dr} = h[T_\infty - T(r_1)]$$

$$k\frac{dT(r_2)}{dr} = \dot{q}_s$$

$$\dot{q}_s = \frac{\dot{Q}_s}{A_2} = \frac{\dot{Q}_s}{2\pi r_2 L} = \frac{0.85 \times 300\text{ W}}{2\pi(0.04\text{ m})(6\text{ m})} = 169.1\text{ W/m}^2$$

$$r\frac{dT}{dr} = C_1$$

$$\frac{dT}{dr} = \frac{C_1}{r}$$

$$T(r) = C_1 \ln r + C_2$$

$$r = r_2: \qquad k\frac{C_1}{r_2} = \dot{q}_s \;\rightarrow\; C_1 = \frac{\dot{q}_s r_2}{k}$$

$$r = r_1: \qquad -k\frac{C_1}{r_1} = h[T_\infty - (C_1 \ln r_1 + C_2)] \;\rightarrow\; C_2 = T_\infty - \left(\ln r_1 - \frac{k}{hr_1}\right)C_1 = T_\infty - \left(\ln r_1 - \frac{k}{hr_1}\right)\frac{\dot{q}_s r_2}{k}$$

$$T(r) = C_1 \ln r + T_\infty - \left(\ln r_1 - \frac{k}{hr_1}\right)C_1 = T_\infty + \left(\ln r - \ln r_1 + \frac{k}{hr_1}\right)C_1 = T_\infty + \left(\ln\frac{r}{r_1} + \frac{k}{hr_1}\right)\frac{\dot{q}_s r_2}{k}$$

$$= -10°\text{C} + \left(\ln\frac{r}{r_1} + \frac{14\text{ W/m}\cdot°\text{C}}{(30\text{ W/m}^2\cdot°\text{C})(0.037\text{ m})}\right)\frac{(169.1\text{ W/m}^2)(0.04\text{ m})}{14\text{ W/m}\cdot°\text{C}} = -10 + 0.483\left(\ln\frac{r}{r_1} + 12.61\right)$$

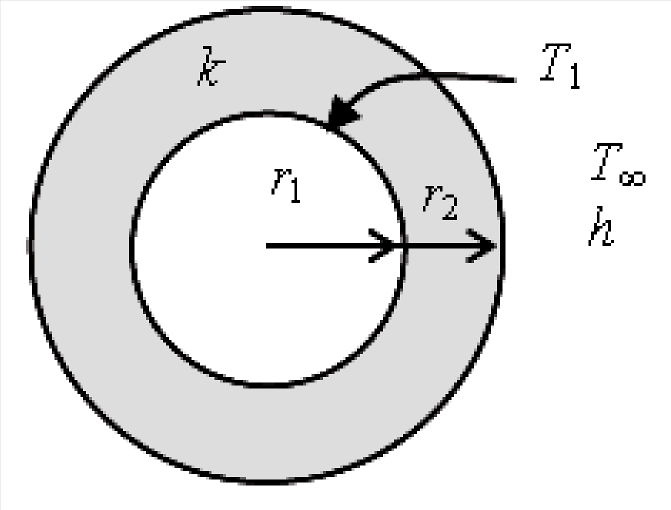
Inner surface ($r = r_1$): $T(r_1) = -10 + 0.483\left(\ln\frac{r_1}{r_1} + 12.61\right) = -10 + 0.483(0 + 12.61) = -\mathbf{3.91°C}$

Outer surface ($r = r_2$): $T(r_1) = -10 + 0.483\left(\ln\frac{r_2}{r_1} + 12.61\right) = -10 + 0.483\left(\ln\frac{0.04}{0.037} + 12.61\right) = -\mathbf{3.87°C}$

Note that the pipe is essentially isothermal at a temperature of about -3.9°C.

Problem 2

A spherical container of inner radius r_1=2 m, outer radius r_2=2.1 m, and thermal conductivity k=30 W/m·°C is filled with iced water at 0°C. The container is gaining heat by convection from the surrounding air at 25°C with a heat transfer coefficient of h=18 W/m^2·°C. Assuming the inner surface temperature of the container to be 0°C, (a) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the container, (b) obtain a relation for the variation of temperature in the container by solving the differential equation, and (c) evaluate the rate of heat gain to the iced water.



Solution

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0$$

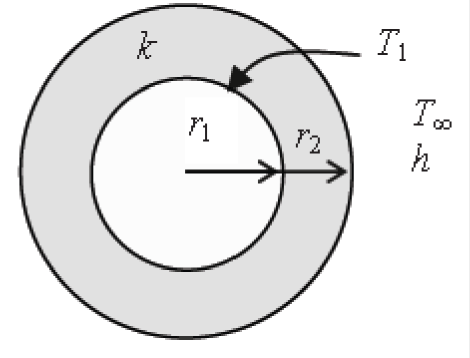
$$T(r_1)=T_1=0°\text{C}$$

$$-k\frac{dT(r_2)}{dr}=h[T(r_2)-T_\infty]$$

$$r^2\frac{dT}{dr}=C_1$$

$$\frac{dT}{dr}=\frac{C_1}{r^2}$$

$$T(r)=-\frac{C_1}{r}+C_2$$



$$r=r_1: \qquad T(r_1)=-\frac{C_1}{r_1}+C_2=T_1$$

$$r=r_2: \qquad -k\frac{C_1}{r_2^2}=h\left(-\frac{C_1}{r_2}+C_2-T_\infty\right)$$

$$C_1=\frac{r_2(T_1-T_\infty)}{1-\frac{r_2}{r_1}-\frac{k}{hr_2}} \quad \text{and} \quad C_2=T_1+\frac{C_1}{r_1}=T_1+\frac{T_1-T_\infty}{1-\frac{r_2}{r_1}-\frac{k}{hr_2}}\frac{r_2}{r_1}$$

$$T(r) = -\frac{C_1}{r} + T_1 + \frac{C_1}{r_1} = C_1\left(\frac{1}{r_1} - \frac{1}{r}\right) + T_1 = \frac{T_1 - T_\infty}{1 - \frac{r_2}{r_1} - \frac{k}{hr_2}}\left(\frac{r_2}{r_1} - \frac{r_2}{r}\right) + T_1$$

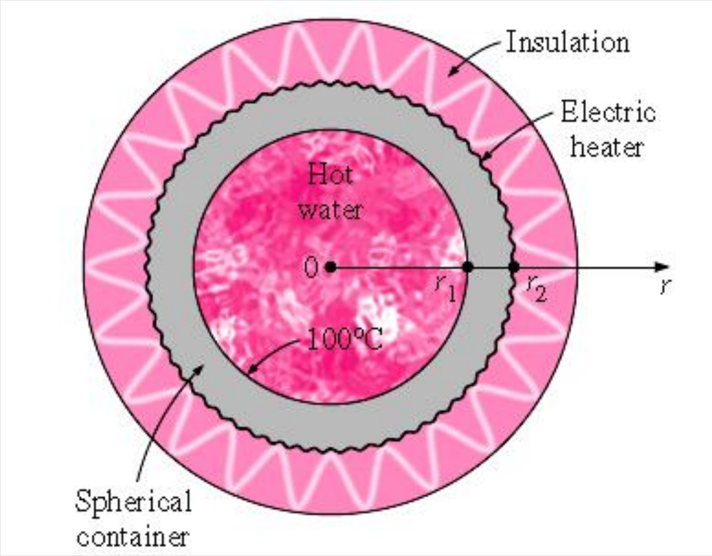
$$= \frac{(0 - 25)°\text{C}}{1 - \frac{2.1}{2} - \frac{30\ \text{W/m}\cdot°\text{C}}{(18\ \text{W/m}^2\cdot°\text{C})(2.1\,\text{m})}}\left(\frac{2.1}{2} - \frac{2.1}{r}\right) + 0°\text{C} = 29.63(1.05 - 2.1/r)$$

$$\dot{Q} = -kA\frac{dT}{dr} = -k(4\pi r^2)\frac{C_1}{r^2} = -4\pi k C_1 = -4\pi k\,\frac{r_2(T_1 - T_\infty)}{1 - \frac{r_2}{r_1} - \frac{k}{hr_2}}$$

$$= -4\pi(30\ \text{W/m}\cdot°\text{C})\frac{(2.1\,\text{m})(0 - 25)°\text{C}}{1 - \frac{2.1}{2} - \frac{30\ \text{W/m}\cdot°\text{C}}{(18\ \text{W/m}^2\cdot°\text{C})(2.1\,\text{m})}} = \mathbf{23{,}460\ W}$$

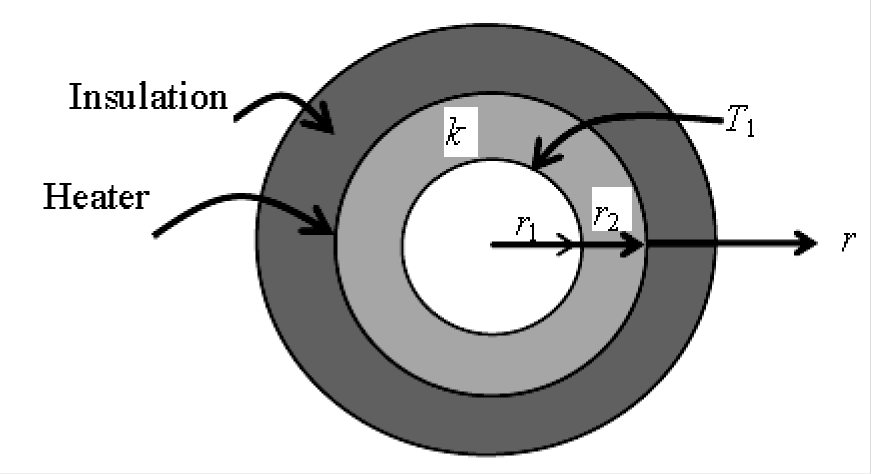
Problem 3

In a food processing facility, a spherical container of inner radius r_1=40 cm, outer radius r_2=41 cm, and thermal conductivity k=1.5 W/m·°C is used to store hot water and to keep it at 100°C at all times. To accomplish this, the outer surface of the container is wrapped with a 500-W electric strip heater and then insulated. The temperature of the inner surface of the container is observed to be nearly 100°C at all times. Assuming 10 percent of the heat generated in the heater is lost through the insulation, (a) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the container, (b) obtain a relation for the variation of temperature in the container material by solving the differential equation, and (c) evaluate the outer surface temperature of the container. Also determine how much water at 100°C this tank can supply steadily if the cold water enters at 20°C.



Solution

Properties The thermal conductivity is given to be $k = 1.5$ W/m·°C. The specific heat of water at the average temperature of (100+20)/2 = 60°C is 4.185 kJ/kg·°C (Table A-9).



$$\dot{q}_s = \frac{\dot{Q}_s}{A_2} = \frac{\dot{Q}_s}{4\pi r_2^2} = \frac{0.90 \times 500 \text{ W}}{4\pi(0.41 \text{ m})^2} = 213.0 \text{ W/m}^2$$

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$$

$$T(r_1) = T_1 = 100°\text{C}$$

$$k\frac{dT(r_2)}{dr} = \dot{q}_s$$

$$r^2 \frac{dT}{dr} = C_1$$

$$\frac{dT}{dr} = \frac{C_1}{r^2}$$

$$T(r) = -\frac{C_1}{r} + C_2$$

$$r = r_2: \qquad k\frac{C_1}{r_2^2} = \dot{q}_s \;\rightarrow\; C_1 = \frac{\dot{q}_s r_2^2}{k}$$

$$r = r_1: \qquad T(r_1) = T_1 = -\frac{C_1}{r_1} + C_2 \;\rightarrow\; C_2 = T_1 + \frac{C_1}{r_1} = T_1 + \frac{\dot{q}_s r_2^2}{k r_1}$$

$$T(r) = -\frac{C_1}{r} + C_2 = -\frac{C_1}{r} + T_1 + \frac{C_1}{r_1} = T_1 + \left(\frac{1}{r_1} - \frac{1}{r}\right)C_1 = T_1 + \left(\frac{1}{r_1} - \frac{1}{r}\right)\frac{\dot{q}_s r_2^2}{k}$$

$$= 100°\text{C} + \left(\frac{1}{0.40\text{ m}} - \frac{1}{r}\right)\frac{(213\text{ W/m}^2)(0.41\text{ m})^2}{1.5\text{ W/m}\cdot°\text{C}} = 100 + 23.87\left(2.5 - \frac{1}{r}\right)$$

Outer surface ($r = r_2$): $T(r_2) = 100 + 23.87\left(2.5 - \frac{1}{r_2}\right) = 100 + 23.87\left(2.5 - \frac{1}{0.41}\right) = \mathbf{101.5°C}$

$$\dot{Q} = \dot{m}c_p\Delta T \;\rightarrow\; \dot{m} = \frac{\dot{Q}}{c_p\Delta T} = \frac{0.450\text{ kJ/s}}{(4.185\text{ kJ/kg}\cdot°\text{C})(100-20)°\text{C}} = 0.00134\text{ kg/s} = \mathbf{4.84\,kg/h}$$